Cosmology and the Shape of the Universe

Cosmology is the study of the entire universe. Specifically it is the study of the origin, behavior, and eventual fate of the entire universe. We can treat cosmology in a scientific way because of the parallel development of two sciences: Physics and Astronomy. In astronomy techniques have been developed to look farther and farther into space, seeing more and more of the entire universe at once. In physics we have General Relativity and Particle Physics. It is by fitting what is seen by astronomers into the equations of General Relativity that we learn about the shape and behavior of the entire universe.

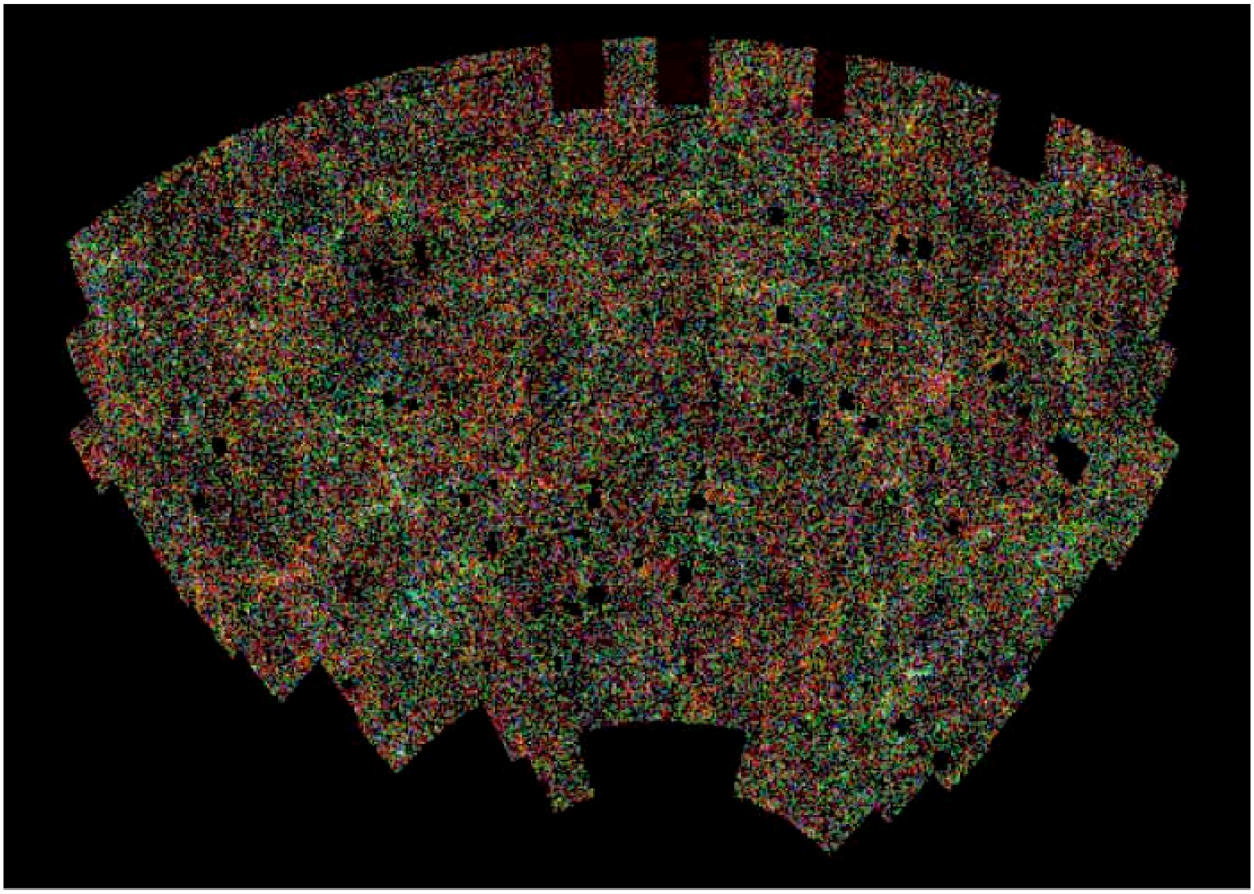
The Shape of the Entire Universe

We have seen how in a curved space two lines that are initially parallel can, without turning, come together by following the curvature of the space. Historically we have interpreted this coming together of world lines without turning as a force (gravity) between the world lines pulling the objects (which form the world lines) together.

There is another implication of letting our spacetime curve. We can imagine that all of spacetime at once may have some shape. It may look like a four-dimensional sphere, or a four-dimensional ellipsoid, or a four-dimensional flat plane. The actual shape of the universe is determined by the distribution of matter and energy. If we look at the distribution of matter in the entire universe we can make rough guesses at what shape the universe must be.

All the stars that you see in the night sky are in a very large (about 2 trillion times as heavy as our sun) system of stars called the Galaxy (with a capitol G). Our Galaxy is one of billions of galaxies that we see beyond the stars. We see these galaxies out to a distance so great that it takes billions of years for the light of these galaxies to reach us from there. By looking at the distribution of these galaxies we can infer the curvature of the space due to these galaxies and so infer the large-scale curvature of spacetime.

As we look out beyond the stars of our own galaxy, we see millions of other galaxies. No matter what direction we look we see galaxies. No matter what direction we look in we see more or less the same number and density of galaxies. This says that whatever shape the universe has it must be more or less the same in all spatial directions. Further, no matter how far away we look we see more or less the same density of galaxies. If we look at our own neighborhood of galaxies we see a certain number of galaxies. If we look farther out (so far out that light takes a billion years to get from there to here) and we look at the same volume of space we see, on average, the same number of galaxies as in our own neighborhood. This says that whatever shape the universe has, the curvature must be more or less the same at all spatially separated points (that is to say at the same time in spacetime).



APM Survey picture of a large part of the sky, about 30 degrees across, showing almost a million galaxies out to a distance of about 2 billion light years.

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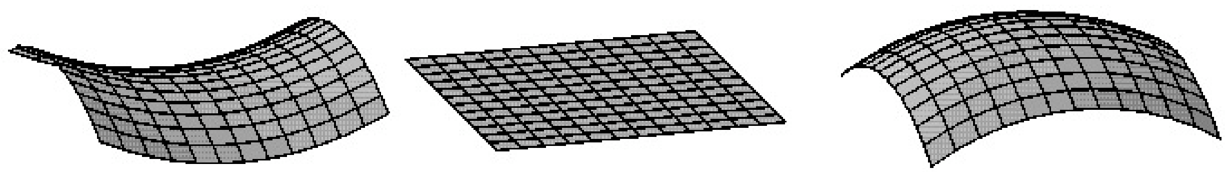
Let's sum this up. By looking at the galaxies we see that on the average they are the same in all directions and they have the same density in ever large volume. Therefore on the scale of the entire universe the curvature must be more or less the same everywhere (on a large scale) and in every direction. This tells us about the curvature in the spatial directions only, because we are looking at the spatial distribution of galaxies. This reasoning gives us no direct information on the curvature in the time direction.

So let's ask what kind of spaces have the same curvature at every point and in every direction. Certainly a flat space satisfies this, as a flat space has zero curvature everywhere, and so it is the same (zero) at every point in every direction. Thus a spacetime that is flat in all spatial directions is a candidate for the shape of our universe.

Are there any others? We can try to visualize a space that has the same curvature everywhere by picturing the path of a car as we drive. Then the curvature of our path is completely determined by how much we turn the steering wheel. If we want the same curvature everywhere we want to turn the wheel by the same amount everywhere. So now picture yourself in your car driving. Turn the steering wheel by a certain amount and hold it there. What path will the car describe? (Try to picture this and answer the question before you read on.) The car will turn and turn and will finally go in an exact circle. A circle is an example of a one dimensional space that has the same curvature everywhere. The two dimensional generalization of the circle is the two-dimensional sphere, which we call the two-sphere. The two-sphere has the same curvature in every direction at every point. The three dimensional generalization of the sphere is called the three-sphere.

Therefore it seems that at any instant in time the three space part of our universe looks like some kind of sphere. This does not mean that it is a round sphere, however. A round sphere (like a globe) is said to have positive curvature. This means that using the usual definition of curvature (=1/radius) a round sphere has the curvature equal to a positive number. Now picture a globe. Let it start to grow in size, letting it's radius get larger and larger. It's curvature then gets smaller and smaller. If we let the radius get infinitely large, the curvature will become infinitely smaller until we have zero curvature--a flat space. The flat plane is a special case of the sphere, one with an infinite radius and zero curvature.

Now what if the radius is a number less than zero? Then the curvature would also be a number less than zero and we would say that we have a sphere of negative curvature. This is actually very hard to visualize. This is not a round sphere--it never curves around and closes on itself. A one dimensional circle of negative curvature is called a hyperbola. A two dimensional version looks somewhat like a saddle shape. Like the plane, the sphere of negative curvature goes on forever and ever, never wrapping back on itself like a sphere of positive curvature does. Like the sphere of positive curvature, this shape generalizes to an arbitrary number of dimensions.



Portions of two-dimensional surfaces of constant curvature. On the left is negative curvature, the middle shows zero curvature (a plane), and the bottom shows positive curvature (a normal sphere).

If you are a mathematician you can prove very rigorously that in an arbitrary number of dimensions the only space that has the same curvature in every dimension at every point is the generalization of the sphere to that number of dimensions.

So at any instant in time, like now, the spatial part of our universe looks, on a large scale, like a three dimensional sphere. This sphere may have positive curvature, negative curvature, or zero curvature (like a flat plane). Of course, since we live in this sphere, we have no hope of visualizing the whole thing.

What about the time direction? We can get hints about the universe in the time direction by trying to look at the state of galaxies in the universe in the past. How can we do this? By simply looking at galaxies that are very far away, because the further they are the longer it took for the light of these galaxies to reach us. We can see galaxies that are so far away that the light from them took billions of years to get here. We are looking at these galaxies as they appeared billions of years in the past.

As we look 10 billion years or less in the past in this way we see that galaxies are more or less the same as they are now. But if we look farther out and so further back in time than ten billion years we see the galaxies start to look different. They become more energetic and brighter. These are the quasars. The quasars are a hint that a long time ago in the past the universe was somewhat different.

Yet when we look in all directions this far out we see more or less the same thing. This implies that even very long ago in the past at any instant in time the spatial part of spacetime was still a three-dimensional sphere (of one of the three types listed above). So how can the shape of the universe be different at different times if at all times the spatial part has the shape of a three dimensional sphere? The only freedom of change left is that the radius of the three dimensional sphere in spacetime can change.

This means that our universe must have the following shape: At every instant in time the spatial part looks like a three dimensional sphere and at different times the radius of the sphere may be different.

Notice that we have decided that the universe must have this shape without once using Einstein's equations of gravitation! We have literally observed the large scale shape of the universe by looking around. Now there is much that we still do not know from this analysis. We do not know if the three-sphere is of positive, negative, or zero curvature. We do not know how (if at all) the curvature of the three-sphere changes with time. To find out about these questions we must use Einstein's equations of gravitation.

Cosmologists mostly make their living modeling the entire universe using Einstein's equations by putting some distribution of matter into the equations and then using the fact that the universe has the shape we described above.

Here is what they find from the theory: The radii of the three-sphere in spacetime is indeed changing and is either steadily getting larger or steadily getting smaller (these are the expanding and contracting universe respectively). The overall behavior of this change in radius is determined by how much matter there is in the universe, which also determines whether the three spheres are of positive, negative, or zero curvature.

If there is enough matter in the universe then the three spheres are of positive curvature. In this case if the radius of these three spheres is getting bigger (if the universe is expanding) then eventually the growth of the radius will stop and the spheres will start to get smaller (the universe will start contracting).

If there is less than a certain amount of matter in the universe then the three-spheres will be of negative curvature and the radius of these spheres will always become larger and larger negative numbers (the universe will expand forever). While the expansion will gradually slow down, it will never stop.

If the amount of matter in the universe is just the right amount then the universe will have zero curvature and will always expand as in the negative curvature case above.

This is what comes out of the Einstein equations. To me it is very surprising that when we look at the amount of matter in the universe (we actually look at the density of matter) it turns out to be very close to the border between the three possibilities. So astronomers are not now able to measure the amount of matter in the universe accurately enough to tell which situation we are now actually in. General relativity itself does not say anything about the state of the universe until we plug in that distribution of matter. We will see that modern unified theories of particle physics when applied to cosmology, however, seem to predict that the universe should have just enough matter to be on the border of the three possibilities. This is called the inflationary universe model and is rather successful in explaining other questions and mysteries left open by general relativity. Inflation was proposed in 1981 by Alan Guth and it still remains to be seen if it will stand the test of time.

In all three possibilities for the shape of the universe if you back up time you find that at some finite time in the past (currently estimated to be about 13.7 billion years ago) all of three-space was concentrated at a point. This is a singularity in the shape of spacetime just like the singularities that are found in black holes. Roger Penrose and Stephen Hawking showed that this singularity must occur no matter what the shape of the universe was in the very far past if general relativity is true. If this is the case then about 13.7 billion years ago all space, time, and matter suddenly appeared (physicists are just starting to play with asking how and from where) and started expanding. This is called the big bang, though it was not an explosion of matter in space as some people have illustrated it. It is true that at the time of the big bang the universe was very hot, but that is because all the matter of the universe was squeezed into a very small space. The universe expanded from this state simply because that is what spacetime does according to Einstein's equations.

I should mention that currently the universe is observed to be expanding.

In the case of positive curvature where the universe eventually starts to contract, spacetime will inevitably return to a singularity.

Physicists do not like this singularity and hope that some theory of quantum gravity will change the situation when space becomes very small.

It is remarkable that modern unified theories of particle physics hint at the possibility that the matter in the universe really did appear from nothing, though that nothing was a far more active and energetic nothing than we usually picture. To quote Guth: "...even if we do not understand the precise scenario, it becomes very plausible that our observed universe emerged from nothing or from almost nothing. I have often heard it said that there is no such thing as a free lunch. It now appears possible that the universe is a free lunch."

The story of the development of matter into atoms and then stars and galaxies is a very interesting (and still somewhat vague one). I will include here a quick summary, and encourage you to find a good book and read about it. I particularly like The First Three Minutes by Stephen Weinberg. There are also some very nice cosmology web sites linked on the class web site.

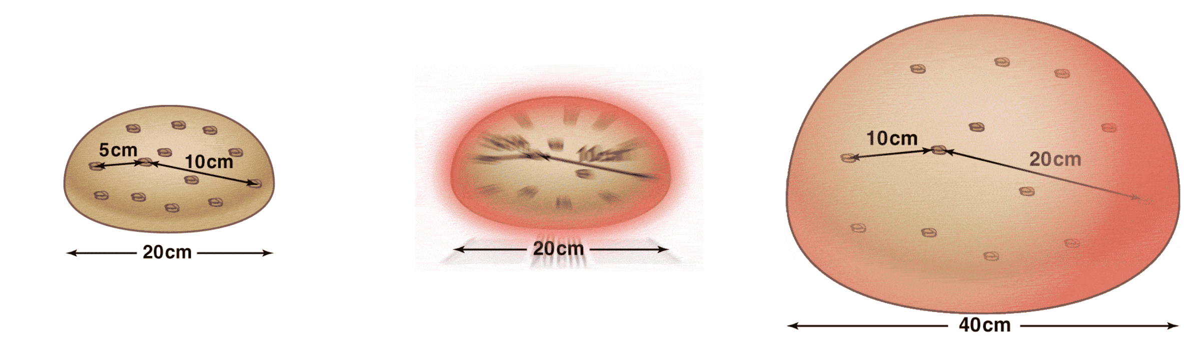
Measuring the Universe

As the universe is observed to be expanding, several questions come immediately into mind:

- How fast is the universe expanding?
- How big is the universe now?
- How long ago was it all at the same place?
- Is the expansion slowing down or speeding up?
- How much matter is there in the universe?

All of these questions are investigated by astronomers.

The first, how fast is the universe expanding, is perhaps the most important one. This is measured by looking at the farthest galaxies. We can measure how fast the galaxies are moving towards or away from us by looking at their light (this light is what is called the redshifted if the galaxy is moving away from us). Then if we know the distance of the galaxy we can infer the rate of the expansion of the universe. We measure the distance of a galaxy mainly by inferring the absolute brightness of that galaxy and comparing this with the apparent brightness of the galaxy in our sky.



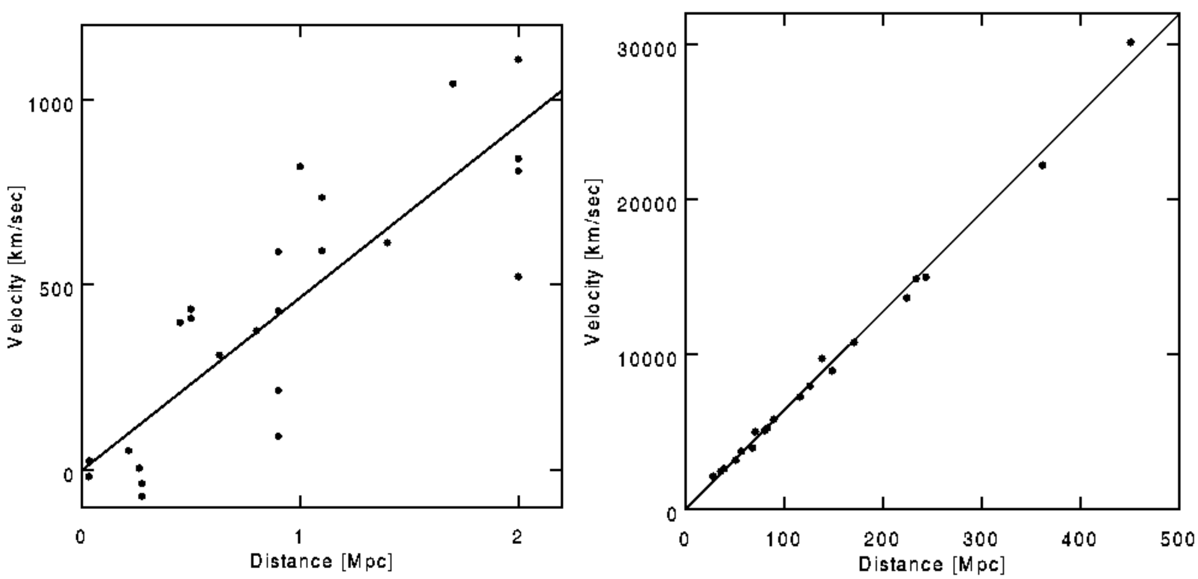
The expansion of the universe illustrated with raisins in an expanding loaf of bread. As the bread cooks, the loaf expands, which will cause the raisins to move apart. Similarly, as the universe expands the galaxies move apart, which makes is look like they are moving away from each other. In an infinite loaf of bread, all the raisins would see the other raisins moving apart, and the further away the raisins are the faster they would seem to be moving. Raisins that are twice are far away seem to be moving twice as fast. In the same way all galaxies would see the same thing: all other galaxies are moving away and the further galaxies are moving away faster.

The measurement of the velocity of the galaxy away from us is rather easy to do accurately. However, as you might expect, inferring the absolute brightness of a galaxy is difficult, especially for galaxies far away (as galaxies go). For this reason the measurement of the rate of expansion of the universe is very difficult to do. Historically, until recently there are two competing measurements which have most of the attention of cosmologists and one was twice as large as the other! Recently, thanks to observations with the Hubble Space Telescope and the Wilkinson Microwave Anisotropy Pobe (WMAP) satellite, a single value is being agreed on. This new value is close to the average of the previous two values.

The rate of expansion of the universe is called the **Hubble Constant**, named for Edwin Hubble who first observed the expansion of the universe. It expresses how fast a galaxy would be moving away from us if it were at a certain distance.

What does this number mean? If we consider the observed universe, then the observed universe will be about one half it's current size about ten billion years ago if Hubble's constant = 50, or about 5 billion years ago if Hubble's constant = 100.

The current value says that, for a galaxy that is 1 megaparsec away (1 megaparsec = 1 million parsecs = 3.262 million light years = 19 million trillion miles), it will be moving away from us at 72 kilometers per second. We say this as “Hubble’s constant is 72 kilometers per second per megaparsec”. There is still an uncertainty of about 10 kilometers per second per megaparsec, so a more truthful statement would be “Hubble’s constant is 72 plus or minus 10 kilometers per second per megaparsec”.



Real data showing the expansion of the universe: on the left are Hubble’s original measurements, and on the right are modern data. The slope of the line is Hubble’s constant, and has changed dramatically as the measurements have improved: Hubble had a slope of about 500, while the modern data shows a slope of about 72.

We can now ask how long ago the observed universe would all be in the same place if we naively backed everything up and squished it all together. If Hubble's constant = 72, this would have happened about 13.7 billion years. Because we think that this actually happened, we call this numbers the age of the universe.

Now all the above analysis assumes that Hubble's constant is really a constant, but this may not be the case. Due to the influence of gravity all the matter in the universe (galaxies, cats, gas, you, me) will be pulling on all the other matter in the universe and this will cause the expansion to slow down. This is not only important for determining the age of the universe more accurately. If there is enough matter, the gravitational pull of the matter will actually cause the expansion to stop and reverse!. This is another perspective on the possibility that the shape of the universe is a 3-sphere of positive curvature. So it is very important to determine how Hubble's constant is changing with time. The quantity that measures this is called the deceleration parameter. This is very hard to measure directly, but there are several indirect measurements which indicate that the universe will expand forever. The value of the deceleration parameter is, however, surprisingly close to the borderline between expanding forever and eventually contracting. This is currently considered a mystery.

There are very recent observations (since 1997) that suggest that the universe is actually accelerating a little bit. The observational evidence for this acceleration is mounting and it looks like a real fact. This is very surprising, because General Relativity, by itself, does not have a mechanism for such an acceleration. But the modern theory of elementary particles does seem to imply such an acceleration when it is put into General Relativity's models of the universe. This is the basis for the inflationary theory described above. The problem is that our current best theories of elementary particles predict an acceleration that is many many orders of magnitude too large. Therefore it is not clear that the observed acceleration can be explained by known theory, so there is a mystery to be solved.

It turns out that one can figure out how much matter there has to be in order to cause the universe to change it's rate of expansion. This gives us a handle on the amount of matter there is in the universe. Also, we can compare the amount of matter we observe (really the density of matter that we observe) by looking at stars etc. with the amount of matter demanded by the observed change in Hubble's constant. What is found is that we do not see enough matter to account for the deceleration parameter that we observe. Further, by watching the motions of galaxies and stars within galaxies it seems that there may be more matter than we observe by looking at stars, etc. The problem of where is this unseen matter is called the missing mass or the dark matter problem. There is about 25 times too little matter visible in stars in galaxies to account for the observed expansion via General Relativity. For this reason physicists have postulated various forms of “dark matter and energy”. Some theories say that this is normal matter that we don't just see, other theories say this is a fundamentally new form of matter. There is no good evidence either way.

To summarize, it is very hard to measure the rate at which the expansion of the universe is changing with time, but it seems to be on the borderline between expanding forever and eventually stopping and contracting.

A Short History of the Beginning of the Universe

If the entire universe is expanding, then if one backs up the expansion (like running the film backwards) one finds that all the matter in the universe will find itself at one point in space at the same time. This is the singularity that was alluded to many pages above and is called the big bang (though it had nothing to do with an explosion). As we look further and further back in time, all the matter in the universe will get more and more squished together. As the matter is more and more squished together, it will get hotter and hotter. To talk of the behavior of matter at these times, one needs to apply modern theories particle physics. This is the first interplay between particle physics and cosmology: Describing the early universe. So what we will do is start at the very beginning and summarize what cosmologists and particle physicists have taught us about the early history of the universe.

Of course, as we go farther and farther back in time the temperature and therefore the energy of the matter in the universe gets higher and higher. Particle physics only knows how to describe matter accurately up to a certain energy or temperature. For higher temperatures (earlier times) than this the predictions become more and more uncertain. In particular, at times earlier than about a second after the big bang all the descriptions are tentative, and at times earlier than 10^{-43} seconds after the big bang no one knows how to say anything at all. Therefore this presentation will be split into two sections, from 10^{-43} seconds to a second after the big bang (the uncertain part), and then from a second onward (the more certain part). In part this description will use material about elementary particles and their behavior. Though this is outside the scope of this class, I mention the effects so that they may connect with any reading you do on particle physics.

Here is a sample history of the universe. The numbers here are for illustration only because they can depend sensitively on the details of the theory, many of which are still uncertain. In particular, these numbers do not match the current best estimates and do not include recent results on dark matter, dark energy and possible acceleration (someday I'll compile a more up-to-date history). I assume the current size of the observed universe to be 10 billion light years, and a Hubble constant of 50 (as opposed to the current best guess of 72).

Time After Bang	Relative Size	Size of Observed Universe	Temperature (in Degrees Kelvin)	Density (in grams per cubic cm)

Very uncertain part:

10^{-43} s	$5x10^{-31}$	.00186 in.	10^{31}	$5x10^{93}$

No one knows what things were like before this.
This is the era where grand unified theories were probably important. This means that: there would have only been one force between the particles besides gravity; electrons, quarks and neutrinos would have been indistinguishable; all particles are continuously created and annihilated copiously as matter-antimatter pairs. Ends with more matter than anti-matter, and this is when the inflationary phase of the universe may have occurred.

10^{-34} s	$1.6x10^{-26}$	5 feet	$3x10^{26}$	10^{90}

This is the era when the strong nuclear force and the electroweak force first appear as different entities. Thus now there is a difference between quarks and the other particles. Both of these forces would appear as long range forces, in particular the quarks would be free particles and there would be no difference between electrons and neutrinos.

10^{-12} s $5x10^{-15}$ 294 million miles 10^{15} 10^{26}

Now the electroweak force breaks into the electromagnetic force and the weak nuclear force, so now there is a difference between electrons and neutrinos. This is presumably the time when particles acquired mass. Particles are still being created and annihilated copiously as matter-antimatter pairs.

10^{-6} s $1.6x10^{-12}$ 93 billion miles $5x10^{12}$ 10^{15}

Now the quarks bind together to form hadrons, because of the drop in the temperature of the universe.

More certain part:

1 s $1.6x10^{-9}$ 15.8 light years (ly) 10^{10} $5x10^{5}$

The temperature of the universe is now low enough that the heavier quarks can no longer be created and annihilated copiously. Now the stable matter in the universe completely consists of electrons, neutrinos, photons, positrons, neutrons and protons. Electron and positron pairs are still being created and annihilated copiously as matter-antimatter pairs. The neutrinos now stop interacting with matter, and neutrons and protons are turning into each other at a very high rate.

All this time the universe has been opaque because it is full of charged particles which interact readily with light.

1.5 s $2x10^{-9}$ 20 light years

Neutron and proton interchanging stops, leaving about six protons for every neutron.

4 s $3.2x10^{-9}$ 32 light years $5x10^{9}$ $3x10^{4}$

The universe is now about as hot as the center of a star. Electron-positron pair creation stops, and all leftover positrons find electrons to annihilate with, leaving relatively few electrons. There are more electrons than positrons because of the state of things at the end of the grand unification time above.

3 min $1.2x10^{-8}$ 212 light years 10^{9}

The neutrons and protons have now slowed down enough to start binding together. This is known as nucleosynthesis and ended with about all the neutrons bound with protons in either helium or deuterium nuclei. The rest of the protons are alone as hydrogen nuclei. This ends with about 4 protons for every helium nucleus. No appreciable amount of heavier nuclei are formed.

400,000 yr $5.6x10^{-3}$ 56 million ly   4000

The electrons now slow down enough to be captured by the hydrogen, deuterium, and helium nuclei. This is known as recombination, even though it is the first time that these particles have combined. Thus at this point the first stable atoms were formed. As most of the electrons and nuclei combine to form uncharged atoms which do not interact so much with light, the universe first becomes transparent at this point, leaving a glow that has now been observed (as the 2.7 degree Kelvin background radiation). This is when the gravitational force between the atoms first made itself felt and the first star formation may have begun at this time.

1 million yr 0.009 88 million ly

Almost everything is now hydrogen and helium gas, with perhaps the first stars, and the beginnings of galaxy formation. It would be very dark.

1 billion yr 0.28 2.8 billion ly

Galaxy formation well under way.

7 billion yr 0.74 7.4 billion ly

Solar System forms.

12.7 billion yr 1.0 10 billion ly

Course taught on general relativity and cosmology at the California academy of Sciences in San Francisco on Earth in the Milky Way Galaxy in the Local Group in the Virgo Supercluster of galaxies. For some 60 years humans have known how to investigate the history of the universe.

Using the current best numbers, the current age of the universe is actually 13.7 billion years.

Remaining Questions

The above account explains much:

- The observed relative abundance of hydrogen and helium.
- The observed background radiation that is seen in all directions.
- The observed expansion of the universe.
- The observed homogeneity of the universe.

There are several questions that remain from the above account which we will turn to in this class:

- Why is there the matter that there is--one would have expected exactly as much matter as antimatter created in pairs so that it would all have disappeared when the pair creation ceased.
- Where did all the energy to make the matter-antimatter pairs come from in the first place?
- Why is the deceleration parameter that describes the change in the rate of expansion of the universe so close to the borderline between expanding forever and eventually stopping and contracting?
- What is dark matter and energy?

Then, of course, there is the question of how galaxies formed. There has been a lot of recent progress on this question primarily through observations by the WMAP satellite, which has detected the right amount of very small inhomogeneities in the radiation from the big bang to account for the eventual formation of galaxies through gravity.